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## Almost contra $v$ -open mappings

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### ABSTRACT

The aim of this paper is to introduce and study the concept of almost contra- $v$ -open mappings and the interrelationship between other almost contra-open maps.

**Keywords:**  $v$ -open set,  $v$ -open map,  $v$ -closed map, Almost contra-open map, Almost contra-pre open map and Almost contra  $v$ -open map.

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### 1. INTRODUCTION

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Open mappings are one such mappings which are studied for different types of open sets by various mathematicians for the past many years. N.Biswas, discussed about semiopen mappings in the year 1970, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defined and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced  $\alpha$ -open and  $\alpha$ -closed mappings in the year in 1983, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-closed mappings in the year 1989 and G.B.Navalagi further verified few results about semipreclosed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced  $\beta$ -open mappings in the year 1983 and Saeid Jafari and T.Noiri, studied about  $\beta$ -closed mappings in the year 2000. C. W. Baker, introduced Contra-open functions and contra-closed functions in the year 1997. M.Caldas and C.W.Baker introduced contra pre-semiopen Maps in the year 2000. In the year 2010, S. Balasubramanian and P.A.S.Vyjayanthi introduced  $v$ -open mappings and in the year 2011 they further defined almost  $v$ -open mappings. In the last year S. Balasubramanian and P.A.S.Vyjayanthi introduced  $v$ -closed and Almost  $v$ -closed mappings. Inspired with these concepts and its interesting properties we in this paper tried to study a new variety of open maps called contra  $v$ -open maps. Throughout the paper  $X$ ,  $Y$  means topological spaces  $(X, \tau)$  and  $(Y, \sigma)$  on which no separation axioms are assured.

### 2. PRELIMINARIES

**Definition 2.1:**  $A \subseteq X$  is said to be

- regular open[pre-open; semi-open;  $\alpha$ -open;  $\beta$ -open] if  $A = \text{int}(\text{cl}(A))$  [ $A \subseteq \text{int}(\text{cl}(A))$ ;  $A \subseteq \text{cl}(\text{int}(A))$ ;  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ ;  $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ ] and regular closed[pre-closed; semi-closed;  $\alpha$ -closed;  $\beta$ -closed] if  $A = \text{cl}(\text{int}(A))$  [ $\text{cl}(\text{int}(A)) \subseteq A$ ;  $\text{int}(\text{cl}(A)) \subseteq A$ ;  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ ;  $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$ ]
- $v$ -open if there exists regular-open set  $U$  such that  $U \subseteq A \subseteq \text{cl}(U)$ .
- $g$ -closed[ $rg$ -closed] if  $\text{cl}(A) \subseteq U$  [ $\text{rc}(\text{cl}(A)) \subseteq U$ ] whenever  $A \subseteq U$  and  $U$  is open[ $rg$ -open] in  $X$  and  $g$ -open[ $rg$ -open] if its complement  $X - A$  is  $g$ -closed[ $rg$ -closed].

**Remark 1:** We have the following implication diagrams for open sets.

$$\begin{array}{ccccccc} r\text{-open set} & \rightarrow & \alpha\text{-open set} & \rightarrow & v\text{-open set} & & \\ \downarrow & & & & \downarrow & & \\ \text{pre-open set} & \leftarrow & \text{open set} & \rightarrow & \alpha\text{-open set} & \rightarrow & \text{semi-open set} \rightarrow \beta\text{-open set.} \end{array}$$

**Definition 2.2:** A function  $f: X \rightarrow Y$  is said to be

- continuous [resp: semi-continuous,  $r$ -continuous,  $v$ -continuous] if the inverse image of every open set is open. [resp: semi open, regular open,  $v$ -open].
- irresolute [resp:  $r$ -irresolute,  $v$ -irresolute] if the inverse image of every semi open [resp: regular open,  $v$ -open] set is semi open. [resp: regular open,  $v$ -open].
- closed [resp: semi-closed,  $r$ -closed] if the image of every closed set is closed [resp: semi closed, regular closed].

- d)  $g$ -continuous [resp:  $rg$ -continuous] if the inverse image of every closed set is  $g$ -closed. [resp:  $rg$ -closed].  
 e) contra open [resp: contra semi-open; contra pre-open; contra  $r\alpha$ -open] if the image of every open set in  $X$  is closed [resp: semi-closed; pre-closed;  $r\alpha$ -closed] in  $Y$ .

**Definition 2.3:**  $X$  is said to be  $T_{1/2}[r-T_{1/2}]$  if every (regular) generalized closed set is (regular) closed.

### 3. ALMOST CONTRA $v$ -OPEN MAPPINGS

**Definition 3.1:** A function  $f: X \rightarrow Y$  is said to be Almost contra  $v$ -open if the image of every open set in  $X$  is  $v$ -closed in  $Y$ .

**Theorem 3.1:** Every Almost contra  $r\alpha$ -open map is Almost contra  $v$ -open but not conversely.

**Proof:** Let  $A \subseteq X$  be open  $\Rightarrow f(A)$  is  $r\alpha$ -closed in  $Y$  since  $f: X \rightarrow Y$  is Almost contra  $r\alpha$ -open  $\Rightarrow f(A)$  is  $v$ -closed in  $Y$  since every  $r\alpha$ -closed set is  $v$ -closed. Hence  $f$  is Almost contra  $v$ -open.

**Example 1:** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ ;  $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$ . Let  $f: X \rightarrow Y$  be defined  $f(a) = b$ ,  $f(b) = c$  and  $f(c) = a$ . Then  $f$  is Almost contra  $v$ -open, Almost contra semi-open, Almost contra  $r\alpha$ -open and Almost contra  $\beta$ -open but not Almost contra open, Almost contra pre-open, Almost contra  $r$ -open, Almost contra  $\alpha$ -open and Almost contra  $rp$ -open.

**Example 2:** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\} = \sigma$ . Let  $f: X \rightarrow Y$  be defined  $f(a) = b$ ,  $f(b) = c$  and  $f(c) = a$ . Then  $f$  is not Almost contra  $v$ -open.

**Theorem 3.2:** Every Almost contra  $r$ -open map is Almost contra  $v$ -open but not conversely.

**Proof:** Let  $A \subseteq X$  be open  $\Rightarrow f(A)$  is  $r$ -closed in  $Y$  since  $f: X \rightarrow Y$  is Almost contra  $r$ -open  $\Rightarrow f(A)$  is  $v$ -closed in  $Y$  since every  $r$ -closed set is  $v$ -closed. Hence  $f$  is Almost contra  $v$ -open.

**Theorem 3.3:** Every Almost contra  $v$ -open map is Almost contra semi-open but not conversely.

**Proof:** Let  $A \subseteq X$  be open  $\Rightarrow f(A)$  is  $v$ -closed in  $Y$  since  $f: X \rightarrow Y$  is Almost contra  $v$ -open  $\Rightarrow f(A)$  is semi-closed in  $Y$  since every  $v$ -closed set is semi-closed. Hence  $f$  is Almost contra semi-open.

**Theorem 3.4:** Every Almost contra  $v$ -open map is Almost contra  $\beta$ -open but not conversely.

**Proof:** Let  $A \subseteq X$  be open  $\Rightarrow f(A)$  is  $v$ -closed in  $Y$  since  $f: X \rightarrow Y$  is Almost contra  $v$ -open  $\Rightarrow f(A)$  is  $\beta$ -closed in  $Y$  since every  $v$ -closed set is  $\beta$ -closed. Hence  $f$  is Almost contra  $\beta$ -open.

**Note 1:**

- Almost contra open maps and Almost contra  $v$ -open maps are independent of each other.
- Almost contra  $\alpha$ -open map and Almost contra  $v$ -open map are independent of each other.
- Almost contra pre open map and Almost contra  $v$ -open map are independent of each other.

**Note 2:** We have the following implication diagram among the open maps.

Al.c.  $r$ -open  $\rightarrow$  Al.c.  $r\alpha$ -open  $\rightarrow$  Al.c.  $v$ -open  
 $\downarrow$   $\downarrow$   
 Al.c. pre-open  $\leftarrow$  Al.c. open  $\rightarrow$  Al.c.  $\alpha$ -open  $\rightarrow$  Al.c.-semi-open  $\rightarrow$  Al.c.  $\beta$ -open.  
 None is reversible.

**Theorem 3.5:** If  $R\alpha C(Y) = RPC(Y)$  then  $f$  is Almost contra  $r\alpha$ -open iff  $f$  is Almost contra  $v$ -open.

**Proof:** Follows from theorem 3.1

Conversely Let  $A \subseteq X$  be open  $\Rightarrow f(A)$  is  $v$ -closed in  $Y$  since  $f: X \rightarrow Y$  is Almost contra  $v$ -open  $\Rightarrow f(A)$  is  $r\alpha$ -closed in  $Y$  since every  $v$ -closed set is  $r\alpha$ -closed. Hence  $f$  is Almost contra  $r\alpha$ -open.

**Theorem 3.6:** If  $RPC(Y) = RC(Y)$  then  $f$  is Almost contra  $r$ -open iff  $f$  is Almost contra  $v$ -open.

**Proof:** Follows from theorem 3.2

Conversely Let  $A \subseteq X$  be open  $\Rightarrow f(A)$  is  $v$ -closed in  $Y$  since  $f: X \rightarrow Y$  is Almost contra  $v$ -open  $\Rightarrow f(A)$  is  $r$ -closed in  $Y$  since every  $v$ -closed set is  $r$ -closed. Hence  $f$  is Almost contra  $r$ -open.

**Theorem 3.7:** If  $RPC(Y) = \alpha C(Y)$  then  $f$  is Almost contra  $\alpha$ -open iff  $f$  is Almost contra  $v$ -open.

**Proof:** Let  $A \subseteq X$  be open  $\Rightarrow f(A)$  is  $\alpha$ -closed in  $Y$  since  $f: X \rightarrow Y$  is Almost contra  $\alpha$ -open  $\Rightarrow f(A)$  is  $v$ -closed in  $Y$  since every  $\alpha$ -closed set is  $v$ -closed. Hence  $f$  is Almost contra  $v$ -open.

Conversely Let  $A \subseteq X$  be open  $\Rightarrow f(A)$  is  $v$ -closed in  $Y$  since  $f: X \rightarrow Y$  is Almost contra  $v$ -open  $\Rightarrow f(A)$  is  $\alpha$ -closed in  $Y$  since every  $v$ -closed set is  $\alpha$ -closed. Hence  $f$  is Almost contra  $\alpha$ -open.

**Theorem 3.8:** If  $f$  is open [ $r$ -open] and  $g$  is Almost contra  $v$ -open then  $g \circ f$  is Almost contra  $v$ -open.

**Proof:** Let  $A \subseteq X$  be open  $\Rightarrow f(A)$  is open in  $Y \Rightarrow g(f(A))$  is  $v$ -closed in  $Z \Rightarrow g \circ f(A)$  is  $v$ -closed in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -open.

**Theorem 3.9:** If  $f$  is open [ $r$ -open] and  $g$  is Almost contra  $r$ -open then  $g \circ f$  is Almost contra  $v$ -open.

**Proof:** Let  $A \subseteq X$  be open  $\Rightarrow f(A)$  is open in  $Y \Rightarrow g(f(A))$  is  $r$ -closed in  $Z \Rightarrow g \circ f(A)$  is  $v$ -closed in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -open.

**Theorem 3.10:** If  $f$  is open [ $r$ -open] and  $g$  is Almost contra  $r\alpha$ -open then  $g \circ f$  is Almost contra  $v$ -open.

**Proof:** Let  $A \subseteq X$  be open in  $X \Rightarrow f(A)$  is open in  $Y \Rightarrow g(f(A))$  is  $r\alpha$ -closed in  $Z \Rightarrow g(f(A))$  is  $v$ -closed in  $Z \Rightarrow g \circ f(A)$  is  $v$ -closed in  $Z$ . Hence  $g \circ f$  is almost Almost contra  $v$ -open.

**Corollary 3.1:**

- If  $f$  is open [ $r$ -open] and  $g$  is Almost contra  $v$ -open then  $g \circ f$  is Almost contra semi-open and hence Almost contra  $\beta$ -open.
- If  $f$  is open [ $r$ -open] and  $g$  is Almost contra  $r$ -open then  $g \circ f$  is Almost contra semi-open and hence Almost contra  $\beta$ -open.

c) If  $f$  is open[ $r$ -open] and  $g$  is Almost contra  $ra$ -open then  $g \circ f$  is Almost contra semi-open and hence Almost contra  $\beta$ -open.

**Theorem 3.14:** If  $f$  is Almost contra open[Almost contra- $r$ -open] and  $g$  is  $v$ -closed then  $g \circ f$  is Almost contra- $v$ -open.

**Proof:** Let  $A \subseteq X$  be open in  $X \Rightarrow f(A)$  is closed in  $Y \Rightarrow g(f(A))$  is  $v$ -closed in  $Z \Rightarrow g \circ f(A)$  is  $v$ -closed in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -open.

**Theorem 3.15:** If  $f$  is Almost contra open[Almost contra- $r$ -open] and  $g$  is  $r$ -closed then  $g \circ f$  is Almost contra- $v$ -open.

**Proof:** Let  $A \subseteq X$  be open in  $X \Rightarrow f(A)$  is closed in  $Y \Rightarrow g(f(A))$  is  $r$ -closed in  $Z \Rightarrow g(f(A))$  is  $v$ -closed in  $Z \Rightarrow g \circ f(A)$  is  $v$ -closed in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -open.

**Theorem 3.16:** If  $f$  is Almost contra open[Almost contra- $r$ -open] and  $g$  is  $ra$ -closed then  $g \circ f$  is Almost contra- $v$ -open.

**Proof:** Let  $A \subseteq X$  be open in  $X \Rightarrow f(A)$  is closed in  $Y \Rightarrow g(f(A))$  is  $ra$ -closed in  $Z \Rightarrow g(f(A))$  is  $v$ -closed in  $Z \Rightarrow g \circ f(A)$  is  $v$ -closed in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -open.

**Corollary 3.2:**

a) If  $f$  is Almost contra open[Almost contra- $r$ -open] and  $g$  is  $v$ -closed then  $g \circ f$  is Almost contra-semi-open and hence Almost contra  $\beta$ -open.

b) If  $f$  is Almost contra open[Almost contra- $r$ -open] and  $g$  is  $r$ -closed then  $g \circ f$  is Almost contra-semi-open and hence Almost contra  $\beta$ -open.

c) If  $f$  is Almost contra open[Almost contra- $r$ -open] and  $g$  is  $ra$ -closed then  $g \circ f$  is Almost contra-semi-open and hence Almost contra  $\beta$ -open.

**Theorem 3.20:** If  $f: X \rightarrow Y$  is Almost contra  $v$ -open then  $v(\overline{f(A)}) \subseteq f(\bar{A})$

**Proof:** Let  $A \subseteq X$  and  $f: X \rightarrow Y$  be Almost contra  $v$ -open. Then  $f(\bar{A})$  is  $v$ -closed in  $Y$  and  $f(A) \subseteq f(\bar{A})$ . This implies  $v(\overline{f(A)}) \subseteq v(f(\bar{A}))$

$\rightarrow$  (1)  
 Since  $f(\bar{A})$  is  $v$ -open in  $Y$ ,  $v(f(\bar{A})) = f(\bar{A}) \rightarrow$  (2)

Using (1) & (2) we have  $v(\overline{f(A)}) = f(\bar{A})$  for every subset  $A$  of  $X$ .

**Remark 2:** Converse is not true in general.

**Corollary 3.3:** If  $f: X \rightarrow Y$  is Almost contra  $r$ -open then  $v(\overline{f(A)}) \subseteq f(\bar{A})$ .

**Theorem 3.21:** If  $f: X \rightarrow Y$  is Almost contra  $v$ -open and  $A \subseteq X$  is open,  $f(A)$  is  $\tau_v$ -closed in  $Y$ .

**Proof:** Let  $A \subseteq X$  and  $f: X \rightarrow Y$  be Almost contra  $v$ -open  $\Rightarrow v(\overline{f(A)}) \subseteq f(\bar{A})$  (by theorem 3.20.)  $\Rightarrow v(\overline{f(A)}) \subseteq f(A)$  since  $f(A) = f(\bar{A})$  as  $A$  is open. But  $f(A) \subseteq v(\overline{f(A)})$ . Therefore we have  $f(A) = v(\overline{f(A)})$ . Hence  $f(A)$  is  $\tau_v$ -closed in  $Y$ .

**Corollary 3.4:** If  $f: X \rightarrow Y$  is Almost contra  $r$ -open, then  $f(A)$  is  $\tau_v$ -closed in  $Y$  if  $A$  is  $r$ -open set in  $X$ .

**Theorem 3.22:** If  $v(\bar{f(A)}) = r(\bar{f(A)})$  for every  $A \subseteq Y$  then the following are equivalent.

a)  $f: X \rightarrow Y$  is Almost contra  $v$ -open map.

b)  $v(\overline{f(A)}) \subseteq f(\bar{A})$ .

**Proof:** (a)  $\Rightarrow$  (b) follows from theorem 3.20.

(b)  $\Rightarrow$  (a) Let  $A$  be any open set in  $X$ .

Then  $f(A) = f(\bar{A}) \supset v(\overline{f(A)})$  [by hypothesis] We have  $f(A) \subseteq v(\overline{f(A)})$ . Combining these two we have,  $f(A) = v(\overline{f(A)}) = r(\bar{f(A)})$  (by given condition) which implies  $f(A)$  is  $r$ -open and hence  $f(A)$  is  $v$ -closed. Thus for every open set  $A$  in  $X$ , we have  $f(A)$  is  $v$ -closed in  $Y$ . Therefore  $f$  is Almost contra  $v$ -open.

**Theorem 3.23:**  $f: X \rightarrow Y$  is Almost contra  $v$ -open iff for each subset  $S$  of  $Y$  and each closed set  $U$  containing  $f^{-1}(S)$ , there is an  $v$ -open set  $V$  of  $Y$  such that  $S \subseteq V$  and  $f^{-1}(V) \subseteq U$ .

**Proof:** Assume  $f: X \rightarrow Y$  is almost contra  $v$ -open. Let  $S \subseteq Y$  and  $U$  be a closed set of  $X$  containing  $f^{-1}(S)$ . Then  $X-U$  is open in  $X$  and  $f(X-U)$  is  $v$ -closed in  $Y$  as  $f$  is almost contra  $v$ -open and  $V = Y - f(X-U)$  is  $v$ -open in  $Y$ .  $f^{-1}(S) \subseteq U \Rightarrow S \subseteq f(U) \Rightarrow S \subseteq V$  and  $f^{-1}(V) = f^{-1}(Y - f(X-U)) = f^{-1}(Y) - f^{-1}(f(X-U)) = f^{-1}(Y) - (X-U) = X - (X-U) = U$

Conversely Let  $F$  be open in  $X \Rightarrow F^c$  is closed. Then  $f^{-1}(f(F^c)) \subseteq F^c$ .

By hypothesis there exists an  $v$ -open set  $V$  of  $Y$ , such that  $f(F^c) \subseteq V$  and  $f^{-1}(V) \supset F^c$  and so  $F \subseteq [f^{-1}(V)]^c$ . Hence  $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F) = V^c$ . Thus  $f(F)$  is  $v$ -closed in  $Y$ . Therefore  $f$  is Almost contra  $v$ -open.

**Remark 3:** Composition of two Almost contra  $v$ -open maps is not Almost contra  $v$ -open in general.

**Theorem 3.24:** Let  $X, Y, Z$  be topological spaces and every  $v$ -closed set is open[ $r$ -open] in  $Y$ . Then the composition of two Almost contra  $v$ -open[Almost contra  $r$ -open] maps is Almost contra  $v$ -open.

**Proof:** (a) Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  be Almost contra  $v$ -open maps. Let  $A$  be any open set in  $X$

$\Rightarrow f(A)$  is  $v$ -closed in  $Y \Rightarrow f(A)$  is open in  $Y$  (by assumption)  $\Rightarrow g(f(A))$  is  $v$ -closed in  $Z \Rightarrow g \circ f(A)$  is  $v$ -closed in  $Z$ . Therefore  $g \circ f$  is Almost contra  $v$ -open.

(b) Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  be Almost contra  $v$ -open maps. Let  $A$  be any open set in  $X$

$\Rightarrow f(A)$  is  $r$ -closed in  $Y \Rightarrow f(A)$  is  $v$ -closed in  $Y \Rightarrow f(A)$  is  $r$ -open in  $Y$  (by assumption)  $\Rightarrow f(A)$  is open in  $Y$  (by assumption)  $\Rightarrow g(f(A))$  is  $r$ -closed in  $Z \Rightarrow g \circ f(A)$  is  $v$ -closed in  $Z$ . Therefore  $g \circ f$  is Almost contra  $v$ -open.

**Theorem 3.25:** Let  $X, Y, Z$  be topological spaces and  $Y$  is discrete topological space in  $Y$ . Then the composition of two Almost contra  $v$ -open[Almost contra  $r$ -open] maps is Almost contra  $v$ -open.

**Theorem 3.26:** If  $f: X \rightarrow Y$  is  $g$ -open,  $g: Y \rightarrow Z$  is Almost contra  $v$ -open [Almost contra  $r$ -open] and  $Y$  is  $T_{1/2}$  [ $r$ - $T_{1/2}$ ] then  $g \circ f$  is Almost contra  $v$ -open.

**Proof:** (a) Let  $A$  be an open set in  $X$ . Then  $f(A)$  is  $g$ -open set in  $Y \Rightarrow f(A)$  is open in  $Y$  as  $Y$  is  $T_{1/2} \Rightarrow g(f(A))$  is  $v$ -closed in  $Z$  since  $g$  is Almost contra  $v$ -open  $\Rightarrow g \circ f(A)$  is  $v$ -closed in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -open.  
(b) Let  $A$  be an open set in  $X$ . Then  $f(A)$  is  $g$ -open set in  $Y \Rightarrow f(A)$  is open in  $Y$  as  $Y$  is  $T_{1/2} \Rightarrow g(f(A))$  is  $r$ -closed in  $Z$  since  $g$  is Almost contra  $r$ -open  $\Rightarrow g \circ f(A)$  is  $v$ -closed in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -open.

**Corollary 3.5:** If  $f: X \rightarrow Y$  is  $g$ -closed,  $g: Y \rightarrow Z$  is Almost contra  $v$ -open [Almost contra  $r$ -open] and  $Y$  is  $T_{1/2}$  [ $r$ - $T_{1/2}$ ] then  $g \circ f$  is Almost contra  $p$ -open and hence Almost contra  $\beta$ -open.

**Theorem 3.27:** If  $f: X \rightarrow Y$  is  $rg$ -closed,  $g: Y \rightarrow Z$  is Almost contra  $v$ -open [Almost contra  $r$ -open] and  $Y$  is  $r$ - $T_{1/2}$ , then  $g \circ f$  is Almost contra  $v$ -open.

**Proof:** Let  $A$  be an open set in  $X$ . Then  $f(A)$  is  $rg$ -open in  $Y \Rightarrow f(A)$  is  $r$ -open in  $Y$  since  $Y$  is  $r$ - $T_{1/2} \Rightarrow f(A)$  is open in  $Y$  since every  $r$ -open set is open  $\Rightarrow g(f(A))$  is  $v$ -closed in  $Z \Rightarrow g \circ f(A)$  is  $v$ -closed in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -open.

**Corollary 3.6:** If  $f: X \rightarrow Y$  is  $rg$ -closed,  $g: Y \rightarrow Z$  is Almost contra  $v$ -open [Almost contra  $r$ -open] and  $Y$  is  $r$ - $T_{1/2}$ , then  $g \circ f$  is Almost contra semi-open and hence Almost contra  $\beta$ -open.

**Theorem 3.28:** If  $f: X \rightarrow Y$ ,  $g: Y \rightarrow Z$  be two mappings such that  $g \circ f$  is Almost contra  $v$ -open [Almost contra  $r$ -open] then the following statements are true.

- If  $f$  is continuous [ $r$ -continuous] and surjective then  $g$  is Almost contra  $v$ -open.
- If  $f$  is  $g$ -continuous, surjective and  $X$  is  $T_{1/2}$  then  $g$  is Almost contra  $v$ -open.
- If  $f$  is  $rg$ -continuous, surjective and  $X$  is  $r$ - $T_{1/2}$  then  $g$  is Almost contra  $v$ -open.

**Proof:** (a) Let  $A$  be an open set in  $Y \Rightarrow f^{-1}(A)$  is open in  $X \Rightarrow (g \circ f)(f^{-1}(A))$  is  $v$ -closed in  $Z \Rightarrow g(A)$  is  $v$ -closed in  $Z$ . Hence  $g$  is Almost contra  $v$ -open.

(b) Let  $A$  be an open set in  $Y \Rightarrow f^{-1}(A)$  is  $g$ -open in  $X \Rightarrow f^{-1}(A)$  is open in  $X$  [since  $X$  is  $T_{1/2}$ ]  $\Rightarrow (g \circ f)(f^{-1}(A))$  is  $v$ -closed in  $Z \Rightarrow g(A)$  is  $v$ -closed in  $Z$ . Hence  $g$  is Almost contra  $v$ -open.

(c) Let  $A$  be an open set in  $Y \Rightarrow f^{-1}(A)$  is  $g$ -open in  $X \Rightarrow f^{-1}(A)$  is open in  $X$  [since  $X$  is  $r$ - $T_{1/2}$ ]  $\Rightarrow (g \circ f)(f^{-1}(A))$  is  $v$ -closed in  $Z \Rightarrow g(A)$  is  $v$ -closed in  $Z$ . Hence  $g$  is Almost contra  $v$ -open.

**Corollary 3.7:** If  $f: X \rightarrow Y$ ,  $g: Y \rightarrow Z$  be two mappings such that  $g \circ f$  is Almost contra  $v$ -open [Almost contra  $r$ -open] then the following statements are true.

- If  $f$  is continuous [ $r$ -continuous] and surjective then  $g$  is Almost contra semi-open and hence Almost contra  $\beta$ -open.
- If  $f$  is  $g$  continuous, surjective and  $X$  is  $T_{1/2}$  then  $g$  is Almost contra semi-open and hence Almost contra  $\beta$ -open.
- If  $f$  is  $rg$ -continuous, surjective and  $X$  is  $r$ - $T_{1/2}$  then  $g$  is Almost contra semi-open and hence Almost contra  $\beta$ -open.

**Theorem 3.29:** If  $f: X \rightarrow Y$  is Almost contra  $v$ -open and  $A$  is an open set of  $X$  then  $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$  is Almost contra  $v$ -open.

**Proof:** (a) Let  $F$  be an open set in  $A$ . Then  $F = A \cap E$  for some open set  $E$  of  $X$  and so  $F$  is open in  $X \Rightarrow f(A)$  is  $v$ -closed in  $Y$ . But  $f(F) = f_A(F)$ . Therefore  $f_A$  is Almost contra  $v$ -open.

**Theorem 3.30:** If  $f: X \rightarrow Y$  is Almost contra  $r$ -open and  $A$  is an open set of  $X$  then  $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$  is Almost contra  $v$ -open.

**Proof:** Let  $F$  be an open set in  $A$ . Then  $F = A \cap E$  for some open set  $E$  of  $X$  and so  $F$  is open in  $X \Rightarrow f(A)$  is  $r$ -closed in  $Y \Rightarrow f(A)$  is  $v$ -closed in  $Y$ . But  $f(F) = f_A(F)$ . Therefore  $f_A$  is Almost contra  $v$ -open.

**Corollary 3.8:** If  $f: X \rightarrow Y$  is Almost contra  $v$ -open [Almost contra  $r$ -open] and  $A$  is an open set of  $X$  then  $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$  is Almost contra semi-open and hence Almost contra  $\beta$ -open.

**Theorem 3.31:** If  $f: X \rightarrow Y$  is Almost contra  $v$ -open,  $X$  is  $T_{1/2}$  and  $A$  is  $g$ -open set of  $X$  then  $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$  is Almost contra  $v$ -open.

**Proof:** Let  $F$  be an open set in  $A$ . Then  $F = A \cap E$  for some open set  $E$  of  $X$  and so  $F$  is open in  $X \Rightarrow f(A)$  is  $v$ -closed in  $Y$ . But  $f(F) = f_A(F)$ . Therefore  $f_A$  is Almost contra  $v$ -open.

**Theorem 3.32:** If  $f: X \rightarrow Y$  is Almost contra  $r$ -open,  $X$  is  $T_{1/2}$  and  $A$  is  $g$ -open set of  $X$  then  $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$  is Almost contra  $v$ -open.

**Proof:** Let  $F$  be an open set in  $A$ . Then  $F = A \cap E$  for some open set  $E$  of  $X$  and so  $F$  is open in  $X \Rightarrow f(A)$  is  $r$ -closed in  $Y \Rightarrow f(A)$  is  $v$ -closed in  $Y$ . But  $f(F) = f_A(F)$ . Therefore  $f_A$  is Almost contra  $v$ -open.

**Corollary 3.9:** If  $f: X \rightarrow Y$  is Almost contra  $v$ -open [Almost contra  $r$ -open],  $X$  is  $T_{1/2}$ ,  $A$  is  $g$ -open set of  $X$  then  $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$  is Almost contra semi-open and hence Almost contra  $\beta$ -open.

**Theorem 3.33:** If  $f_i: X_i \rightarrow Y_i$  be Almost contra  $v$ -open [Almost contra  $r$ -open] for  $i = 1, 2$ . Let  $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$  be defined as  $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$ . Then  $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$  is Almost contra  $v$ -open.

**Proof:** Let  $U_1 \times U_2 \subseteq X_1 \times X_2$  where  $U_i$  is open in  $X_i$  for  $i = 1, 2$ . Then  $f(U_1 \times U_2) = f_1(U_1) \times f_2(U_2)$  is  $v$ -closed set in  $Y_1 \times Y_2$ . Then  $f(U_1 \times U_2)$  is  $v$ -closed set in  $Y_1 \times Y_2$ . Hence  $f$  is Almost contra  $v$ -open.

**Corollary 3.10:** If  $f_i: X_i \rightarrow Y_i$  be Almost contra  $v$ -open [Almost contra  $r$ -open] for  $i = 1, 2$ . Let  $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$  be defined as  $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$ , then  $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$  is Almost contra semi-open and hence Almost contra  $\beta$ -open.

**Theorem 3.34:** Let  $h: X \rightarrow X_1 \times X_2$  be Almost contra  $v$ -open. Let  $f_i: X \rightarrow X_i$  be defined as  $h(x) = (x_1, x_2)$  and  $f_i(x) = x_i$ . Then  $f_i: X \rightarrow X_i$  is Almost contra  $v$ -open for  $i = 1, 2$ .

**Proof:** Let  $U_1$  be open in  $X_1$ , then  $U_1 \times X_2$  is open in  $X_1 \times X_2$ , and  $h(U_1 \times X_2)$  is  $v$ -closed in  $X$ . But  $f_1(U_1) = h(U_1 \times X_2)$ , therefore  $f_1$  is Almost contra  $v$ -open. Similarly we can show that  $f_2$  is also Almost contra  $v$ -open and thus  $f_i: X \rightarrow X_i$  is Almost contra  $v$ -open for  $i = 1, 2$ .

**Corollary 3.11:** Let  $h: X \rightarrow X_1 \times X_2$  be Almost contra  $v$ -open. Let  $f_i: X \rightarrow X_i$  be defined as  $h(x) = (x_1, x_2)$  and  $f_i(x) = x_i$ . Then  $f_i: X \rightarrow X_i$  is Almost contra semi-open and hence Almost contra  $\beta$ -open for  $i = 1, 2$ .

#### 4. CONCLUSION

In this paper we introduced the concept of almost contra  $v$ -open mappings, studied their basic properties and the interrelationship between other open maps.

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